7.1 Exponential Growth
7.2 Exponential Decay

$y = a \cdot \underline{b}^x$

"a" determines steepness & reflection across x axis

General Shape...

$f(x) = 3^x$ "b" determines $f(x) = (1/3)^x$

$b > 1$ ← growth

decay → $0 < b < 1$

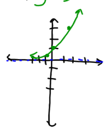
To GRAPH

1st Plot 3 points

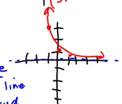
2nd draw asymptotes

3rd use general shape

x	y
−1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$



x	y
−1	$(\frac{1}{3})^{-1} = 3$
0	$(\frac{1}{3})^0 = 1$
1	$(\frac{1}{3})^1 = \frac{1}{3}$



Asymptote
- dashed line
- boundary of graph
- never cross

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Translations of Graphs

$y = a \cdot b^{x-h} + K$

a: steepness, reflection

b: Growth, Decay

h: shifts ← + →−

K: shift ↑ + ↓ −

To Graph

1st Determine Growth/Decay "b"

2nd Take the root (without "h" & "k")

$y = a \cdot b^x$ Plot 3 points

x	y
−1	
0	
1	

3rd shift each point "h" "k"

4th Asymptote $y = $ "k"

5th Domain: $\mathbb{R}$

Range: $+a$ $y > K$; $-a$ $y < K$

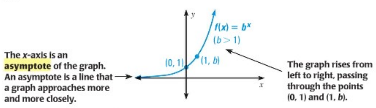
Apr 5-11:40 AM

7.1 Graph Exponential Growth Functions

KEY CONCEPT *For Your Notebook*

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The general shape of the graph of $f(x) = b^x$ is shown below.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Feb 11-7:26 PM

The graph of a function $y = ab^x$ is a vertical stretch or shrink of the graph of $y = b^x$. The y-intercept of the graph of $y = ab^x$ occurs at $(0, a)$ rather than $(0, 1)$.

EXAMPLE 2 **Graph $y = ab^x$ for $b > 1$**

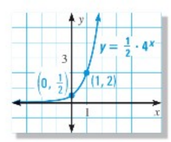
Graph the function.

a. $y = \frac{1}{2} \cdot 4^x$

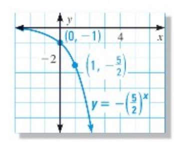
b. $y = -\left(\frac{5}{2}\right)^x$

Solution

a. Plot $\left(0, \frac{1}{2}\right)$ and $(1, 2)$. Then, from *left* to *right*, draw a curve that begins just above the x-axis, passes through the two points, and moves up to the right.



b. Plot $(0, -1)$ and $\left(1, -\frac{5}{2}\right)$. Then, from *left* to *right*, draw a curve that begins just below the x-axis, passes through the two points, and moves down to the right.



Feb 11-7:28 PM

TRANSLATIONS To graph a function of the form $y = ab^{x-h} + k$, begin by sketching the graph of $y = ab^x$. Then translate the graph horizontally by h units and vertically by k units.

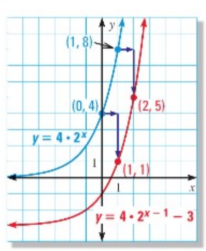
EXAMPLE 3 **Graph $y = ab^{x-h} + k$ for $b > 1$**

Graph $y = 4 \cdot 2^{x-1} - 3$. State the domain and range.

Solution

Begin by sketching the graph of $y = 4 \cdot 2^x$, which passes through $(0, 4)$ and $(1, 8)$. Then translate the graph right 1 unit and down 3 units to obtain the graph of $y = 4 \cdot 2^{x-1} - 3$.

The graph's asymptote is the line $y = -3$. The domain is all real numbers, and the range is $y > -3$.



Feb 11-7:28 PM

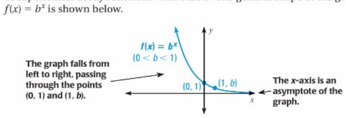
7.2 Graph Exponential Decay Functions

In Lesson 7.1 you studied exponential growth functions. In this lesson, you will study **exponential decay functions**, which have the form $y = ab^x$ where $a > 0$ and $0 < b < 1$. The base b of an exponential decay function is called the **decay factor**.

KEY CONCEPT *For Your Notebook*

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The general shape of the graph of $f(x) = b^x$ is shown below.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

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EXAMPLE 1 **Graph $y = b^x$ for $0 < b < 1$**

Graph $y = \left(\frac{1}{2}\right)^x$.

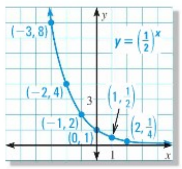
Solution

STEP 1 **Make** a table of values.

x	−3	−2	−1	0	1	2
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

STEP 2 **Plot** the points from the table.

STEP 3 **Draw**, from *right* to *left*, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



Feb 11-7:30 PM

EXAMPLE 2 **Graph $y = ab^x$ for $0 < b < 1$**

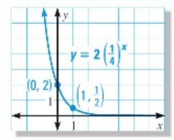
Graph the function.

a. $y = 2\left(\frac{1}{4}\right)^x$

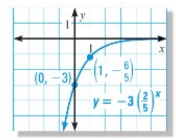
b. $y = -3\left(\frac{2}{5}\right)^x$

Solution

a. Plot (0, 2) and $\left(1, \frac{1}{2}\right)$. Then, from *right* to *left*, draw a curve that begins just above the x-axis, passes through the two points, and moves up to the left.



b. Plot (0, −3) and $\left(1, -\frac{6}{5}\right)$. Then, from *right* to *left*, draw a curve that begins just below the x-axis, passes through the two points, and moves down to the left.



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TRANSFORMATIONS Recall from Lesson 7.1 that the graph of a function $y = ab^x$ is a vertical stretch or shrink of the graph of $y = b^x$, and the graph of $y = ab^{x-h} + k$ is a translation of the graph of $y = ab^x$.

EXAMPLE 3 **Graph $y = ab^{x-h} + k$ for $0 < b < 1$**

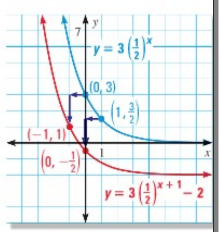
Graph $y = 3\left(\frac{1}{2}\right)^{x+1} - 2$. State the domain and range.

Solution

Begin by sketching the graph of $y = 3\left(\frac{1}{2}\right)^x$, which passes through (0, 3) and $\left(1, \frac{3}{2}\right)$.

Then translate the graph left 1 unit and down 2 units. Notice that the translated graph passes through (−1, 1) and $\left(0, -\frac{1}{2}\right)$.

The graph's asymptote is the line $y = -2$. The domain is all real numbers, and the range is $y > -2$.



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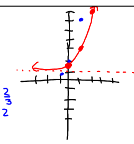
① $f(x) = 2 \cdot 3^{x-1} + 1$

1st. Growth $b = 3$

2nd. $y = 2 \cdot 3^x$

x	y
−1	$2 \cdot 3^{-1} = 2 \cdot \frac{1}{3} = \frac{2}{3}$
0	$2 \cdot 3^0 = 2 \cdot 1 = 2$
1	$2 \cdot 3^1 = 6$



3rd Shift → 1, ↑ 1

4th Asymptote $y = 1$

5th D: $\mathbb{R}$

R: $y > 1$

② $y = -2 \cdot \left(\frac{1}{4}\right)^{x+2} - 1$

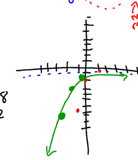
$y = -2\left(\frac{1}{4}\right)^{x+2} - 1$

never reach

Decay

$y = -2\left(\frac{1}{4}\right)^x$

x	y
−1	$-2\left(\frac{1}{4}\right)^{-1} = -2 \cdot 4 = -8$
0	$-2\left(\frac{1}{4}\right)^0 = -2 \cdot 1 = -2$
1	$-2\left(\frac{1}{4}\right)^1 = -\frac{2}{4} = -\frac{1}{2}$



Shift ← 2, ↓ 1

Asymptote $y = -1$

D: $\mathbb{R}$

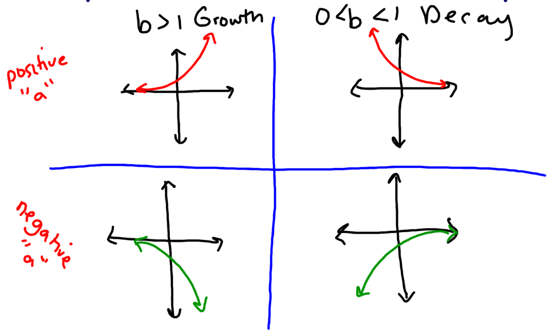
R: $y < -1$

Apr 5-11:47 AM

Goes with Graphing

My chart of Growth/Decay

$b > 1$ Growth | $0 < b < 1$ Decay



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FORMULAS FOR WORD PROBLEMS

EXPONENTIAL GROWTH MODELS When a real-life quantity increases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$\boxed{y = a(1 + r)^t}$$

where a is the initial amount and r is the percent increase expressed as a decimal. Note that the quantity $1 + r$ is the growth factor.

EXPONENTIAL DECAY MODELS When a real-life quantity decreases by a fixed percent each year (or other time period), the amount y of the quantity after t years can be modeled by the equation

$$\boxed{y = a(1 - r)^t}$$

where a is the initial amount and r is the percent decrease expressed as a decimal. Note that the quantity $1 - r$ is the decay factor.

$1 - r$ decay factor

$1 + r$ growth factor

a = initial amount

r = rate

t = time

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Ex ① You bought a dog for \$1000. His value increases 13% each yr. How much is he worth after 7 years.

$y = a(1+r)^t$

$y = 1000(1+.13)^7$

$y = 1000(1.13)^7$

$\boxed{y = \$2352.61}$

② You buy a car for \$13,500. It decreases 22% each year. Whats it worth in 10 years?

$y = a(1-r)^t$

$y = 13500(1-.22)^{10} = \boxed{\$1125.33}$

③ You have a diamond worth \$3500 you bought 15 years ago. If it decreased 17% each year - how much did you pay for it?

$y = a(1-r)^t$

$3500 = a(1-.17)^{15}$

$3500 = a \cdot .061$

$\frac{3500}{.061} = a$ $\boxed{a = \$57,377.05}$

④ College Book originally cost \$1200, that are after 4 hard years are worth \$175.00.

Whats the rate ↓?

$y = a(1-r)^t$

$\frac{175}{1200} = \frac{1200(1-r)^4}{1200}$

$(.145)^{\frac{1}{4}} = \left((1-r)^4\right)^{\frac{1}{4}}$

$.617 = 1 - r$

$-1 \quad -1$

$-.383 = -r$

$r = .383$

$\boxed{38.3\%}$

Apr 5-12:09 PM

EXAMPLE 5 Find the balance in an account

FINANCE You deposit \$4000 in an account that pays 2.92% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. Quarterly

b. Daily

Deposit \$ in an account

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

t: time

r: rate

P: initial Principal amount deposited

n = # of times compounded per year

ⓐ $A = 4000\left(1 + \frac{.0292}{4}\right)^{4 \cdot 1}$

$\boxed{\$4118.09}$

ⓑ $A = 4000\left(1 + \frac{.0292}{365}\right)^{365 \cdot 1}$

$\boxed{\$4118.52}$

Feb 11-7:29 PM

Hw.

Pg 482, #6-21 multiples of 3

Pg 489, #3-6, 9-24 multiples of 3

25 (dont need a calc)

Feb 16-8:19 PM